Estimating water distribution of the rotating sprinkler with pulsating pressure on sloping land

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Abstract: Pulsating pressure plays an important role in improving the poor irrigation quality and the uneven water distribution caused by the terrain slope. Water distribution is one of the key factors in design of the sprinkler irrigation system, however, it is difficult to measure in practice. To provide appropriate technical parameters for the design of sprinkler irrigation system with pulsating pressure on sloping land, a mathematical model was established according to the water conservation principle and finite element idea, and its accuracy was experimentally verified. The model was applied to study the effects of terrain slope, sprinkler arrangement, sprinkler spacing and average pulsating pressure on water distribution on sloping land. The results showed that the water distribution was more favorable under the gentle terrain slope, when slope decreased from 25% to 5%, the uniformity increased from 74.47% to 86.22%. Sprinklers arranged in equilateral triangle and with the spacing close to R_0 had the best water distribution uniformity, the uniformity coefficient (*CU*) of which was 11.43% and 8.75% higher than that in square and rectangular arrangement, respectively. The *CU* increased with the increase of the average pulsating pressure. However, the effect of increasing water pressure on sloping land with pulsating pressure, it is suggested that the sprinkler irrigation systems should be arranged below the terrain slope of 20%, and operated at the average pulsating pressure of 300 kPa. The suitable sprinkler arrangement is the equilateral triangle, and with the spacing of 0.8 R_0 to 1.0 R_0 .

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1 Introduction

Sloping land, as one of the most common topographies in the world, covers about one-fourth of the total farmland area in China. Owing to the poor ability of soil water retention and the application difficulty of the traditional surface irrigation technologies on sloping land, these crops are highly susceptible to drought, which led to low and unstable yields.

Sprinkler irrigation has become one of the most widely used irrigation methods on sloping land because of its good applicability to complex terrain and its ability to control the water quantity and irrigation period^[1,2]. However, affected by the terrain slope, the water distribution uniformity of sprinkler irrigation on sloping land is worse than that on flat land. The water distribution curve on flat land can be regarded as a series of concentric circles formed by the sprinkler in the center, and water application rates at the same distance from the sprinkler are almost the same. However, on sloping land, the water distribution curve is similar to the shape of an "egg", and water application rate of uphill is greater than that of downhill at the same distance from the sprinkler, which resulting in poor water application uniformity^[3].

In the past decades, the adjustment of sprinkler installation angle and height was usually used to increase water distribution uniformity, but it increases the complexity and difficulty in the construction of sprinkler irrigation system^[4,5]. Xiang et al.^[6] has provided a device which can adjust throw radius of sprinkler to improve the water distribution uniformity on sloping land. However, this device was only suitable for terrain slope less than 10%. Furthermore, part-circle sprinklers are often used to match the required coverage area on sloping land, but it increases the water application rates in the wetted area and raises the cost of sprinkler system due to the decrease of sprinkler spacing. Water supply technology with pulsating pressure offers a new method for improving the uniformity of water distribution, and it has been applied in irrigation in recent years^[7-10]. In sprinkler irrigation, the water distribution uniformity and energy distribution can be improved obviously by using pulsating pressure. The Christiansen uniformity coefficient (CU) value of water distribution for a single sprinkler at pulsating pressure was about 10% higher than that of constant pressure. When sprinklers were placed in rectangular arrangement, CU values of water distribution for pulsating pressure were on average 4.06% higher than that for constant pressure with different arrangements. The water distribution pattern is the basic data that was usually used for the design of sprinkler irrigation system^[11-14]. However, limited by experimental conditions, the water distribution pattern on sloping land is too difficult to measure. Calculation of the water distribution pattern becomes an important issue to be resolved in urgent need for the design of sprinkler irrigation systems with pulsating pressure on sloping land.

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The objectives of the paper were to establish a mathematical model for estimating water distribution pattern of a rotating sprinkler with pulsating pressure on sloping land, and the effects of terrain slope, sprinkler arrangement, sprinkler spacing and pulsating water pressure on the water distribution pattern were studied. As a result, the appropriate technical parameter for sprinkler irrigation at pulsating pressure on sloping land was determined.

2 Model construction

2.1 Idea of model construction

Water quantity obtained at different times for any spot in wetted area is different on sloping land, because sprinkler operating pressure changes periodically with time using pulsating pressure to supply water. To calculate the water distribution of a single rotating sprinkler with pulsating pressure on sloping land conveniently, a finite element idea was used to establish the model. Firstly, the wetted area was divided into dozens of micro circular Secondly, the instantaneous water quantity for the sectors. sprinkler nozzle each passing through a micro circular sector on sloping land at pulsating pressure was evaluated by using the radial leg of the measured catch-can data on flat ground at constant pressure, based on the principle of water conservation along the radial direction. Because the duration for nozzle passing through the micro circular sector was very short, resulting in a very small change in sprinkler operating pressure, the pressure for sprinkler running on the micro circular sector could be considered as constant pressure. Thirdly, the water quantity of each micro circular sector was the sum of each rotating period during an irrigation event. Finally, the water distribution of a single rotating sprinkler with pulsating pressure on sloping land was obtained by the combination of water distribution for each micro circular sector. 2.2 Partition of micro circular sectors

The wetted area for a single sprinkler consists of two half ellipses on sloping land at constant pressure as shown in Figure 1. The sprinkler was located at the center of the ellipse. The semi major-axis and semi minor-axis of one-half ellipse is throw radius R on flat ground and uphill throw R_{up} on the uphill slope, respectively. The semi major-axis and semi minor-axis of the other half ellipse is downhill throw R_{down} on the downhill slope and throw radius R on flat ground, respectively. To calculate the water distribution of a single sprinkler with pulsating pressure on sloping land, the wetted area was divided into n micro circular sectors with the same angle according to the rotation of the nozzle.





2.3 Estimating water distribution for a micro circular sector on sloping land

Each instantaneous throw radius for the nozzle passing through the micro circular sectors on sloping land is different, because the sprinkler operates at pulsating pressure. Instantaneous throw radius could be expressed as $R_n(m)$ when the nozzle passes through the n^{th} micro circular sectors for the m^{th} time on sloping land, and the corresponding throw radius on flat ground could be expressed as $R_n'(m)$. The water quantity conversion coefficient $\lambda_n(m)$ was introduced in this study and defined as the ratio of instantaneous throw radius on sloping land to that on flat ground, which can be expressed by:

$$\lambda_n(m) = \frac{R_n(m)}{R_n'(m)} \tag{1}$$

Because water falling within the n^{th} micro circular sectors on sloping land is equal to that on flat ground according to the principle of water conservation along the radial leg, water distribution for the n^{th} micro circular sectors on sloping land can be obtained by $\lambda_n(m)$ and the measured water distribution for the n^{th} micro circular sectors on flat ground.

As shown in Figure 2, M_n is any spot in the n^{th} micro circular sectors on sloping land, and polar coordinates of M_n are (ρ, θ) . The instantaneous water application rate at any spot M_n and instantaneous sprinkler operating pressure for the nozzle passing through the n^{th} micro circular sectors for the m^{th} time on sloping land could be expressed as $P_n(m)$ and $H_n(m)$, respectively. M_n' is the corresponding spot of M_n on flat ground, and its polar coordinates are $\left(\frac{\rho}{\lambda_n(m)}, \theta\right)$. Then, the instantaneous water application rate at any spot M_n for the nozzle passing through the n^{th}

application rate at any spot M_n for the nozzle passing through the n^{th} micro circular sectors for the m^{th} time on sloping land can be estimated by Equation (2).

$$P_{n}(m) = \frac{P_{n}'(m)}{\lambda_{n}(m)} = \frac{P_{n}'(m)R_{n}'(m)}{R_{n}(m)}$$
(2)

where, $P_n'(m)$ is the instantaneous water application rate at the spot M_n' when the nozzle passes through the n^{th} micro circular sectors for the m^{th} time at sprinkler operating pressure of $H_n(m)$ on flat ground.



Figure 2 Diagram of polar coordinates of slope point

The average water application rate at any spot M_n in an irrigation event with pulsating pressure on sloping land can be calculated using Equation (3).

$$P_{n}(m)_{average} = \frac{\sum_{m=1}^{M} P_{n}(m)t_{n}'(m)}{\sum_{n=1}^{N} \sum_{m=1}^{M} t_{n}'(m)} = \frac{\sum_{m=1}^{M} \frac{P_{n}'(m)R_{n}'(m)t_{n}'(m)}{R_{n}(m)}}{\sum_{n=1}^{N} \sum_{m=1}^{M} t_{n}'(m)}$$
(3)

where, $P_n(m)_{average}$ is the average water application rate at any spot M_n in an irrigation event with pulsating pressure on sloping land; $t_n'(m)$ is the sprinkler operating time for the nozzle passing through the n^{th} micro circular sectors for the m^{th} time (s); N is the number of micro circular sectors; M is the sprinkler rotation number in an irrigation event.

Because the average water application rate for any spot in any micro circular sectors can be estimated using Equation (3), the (4)

water distribution for micro circular sectors in an irrigation event with pulsating pressure on sloping land can be obtained.

(1) Calculation of $R_n(m)$ and $R'_n(m)$

 $R_n(m)$ in equation (3) can be calculated as follows ^[15]. a. For uphill slope

. For uprin slope

$$R_n(m) = R'_n(m) \cos \beta [1 - \tan \beta \cot(\theta + \beta)]$$

180° < \theta < 360°

$$R_n(m) = R'_n(m)\cos\beta[1 + \tan\beta\cot(\theta - \beta)]$$

$$0^{\circ} < \theta < 180^{\circ} \tag{5}$$

$$\beta = \pm \frac{180}{\pi} \arctan(i\sin\alpha) \tag{6}$$

where, β is the projected angle (the angle between the projection of water jet trajectory on sloping land and that on level surface), (°); θ is the droplet landing angle on flat ground, (°); *i* is the terrain slope; α is the sprinkler rotating angle, (°), the plus and minus signs are for the term and specify $0^{\circ} \le \alpha \le 180^{\circ}$ and $180^{\circ} \le \alpha < 360^{\circ}$ conditions, respectively.



Figure 3 Sketch of coordinate transformation from flat ground to sloping land

When sprinkler operating pressure is given, and $R_n'(m)$ can be measured on flat ground. Additionally, $R_n'(m)$ can be estimated by Equation (7) ^[16,17].

$$R_n'(m) = 4\mu^2 H_n(m) \sin^2 \gamma \left(\cot \gamma - 0.216 \frac{(H_n(m))^{0.94}}{d} \right)$$
(7)

where, μ is the flow rate coefficient and μ =0.86-0.90^[15]; γ is the nozzle elevation angle, (); *d* is the nozzle diameter, mm.

It is assumed that the function of pulsating pressure is trigonometric. The instantaneous sprinkler operating pressure $H_n(m)$ for the nozzle passing through the n^{th} micro circular sectors for the m^{th} time can be calculated using equation (8).

$$H_n(m) = H_a + A\sin\left(\frac{2\pi t_n(m)}{T}\right)$$
(8)

where, H_a is the average value of pulsating pressure, kPa; A is the amplitude of pulsating pressure (kPa); T is the period of pulsating pressure, s; $t_n(m)$ is the sprinkler operating time before the nozzle passing through the n^{th} micro circular sectors for the m^{th} time, s.

 $t_n(m)$ in Equation (8) can be calculated using Equation (9).

$$t_n(m) = \sum_{m=1}^{M} \sum_{n=1}^{N} t'_n(m)$$
(9)

 $t_n'(m)$ in Equation (9) can be calculated using Equation (10).

$$t_{n}'(m) = \frac{T_{n}'(m)}{N}$$
(10)

where, $T_n'(m)$ is the sprinkler operating time per rotation at pressure of $H_n(m)$, s. The relationship between $T_n'(m)$ and $H_n(m)$ was fitted by the measured data.

(2) Calculation of $P_n'(m)$

The relationship between instantaneous water application rate and average water application rate at the spot M_n' for rotating sprinklers could be expressed as Equation (11).

$$P_{n}'(m) = \frac{P_{n}'(m)_{average} T_{n}'(m)}{t_{n}'(m)}$$
(11)

where, $P_n'(m)_{average}$ is the average water application rate at the spot M_n' under sprinkler operating pressure of $H_n(m)$ on flat ground.

 $P_n'(m)_{average}$ could be calculated by the measured catch-can data on flat ground. Interpolations are performed within the experimentally-obtained catch-can profiles (the radial leg of the catch-cans data measured on flat ground at constant pressure setting for all tests) according to distance from the sprinkler and the sprinkler operating pressure. Thus, there are two interpolations: firstly, along the radial leg of the measured catch-can data; Secondly, between two adjacent pressures from the experimental data.

Linear interpolation is used to obtain the value of $P_n'(m)_{average}$ at the spot M_n' from the experimental data. Because polar coordinates of M_n' are $\left(\frac{\rho}{\lambda_n(m)}, \theta\right)$, the distance from M_n' to the sprinkler is $\frac{\rho}{\lambda_n(m)}$. The average water application rate at the

distance of $\frac{\rho}{\lambda_n(m)}$ from the sprinkler and the operating pressure

of H_1 can be calculated by Equation (12). H_1 is the designated pressure at the sprinkler in experiment which is known. Suppose that $d_1 \leq \frac{\rho}{1-\rho} \leq d_2$.

$$t \quad d_1 \le \frac{\gamma}{\lambda_n(m)} \le d_2 .$$

$$P_{11}' = \frac{P_1 - P_2}{d_1 - d_2} \frac{\rho}{\lambda_n(m)} + \frac{P_2 d_1 - P_1 d_2}{d_1 - d_2}$$
(12)

where, P_{11} ' is the average water application rate at the distance of $\frac{\rho}{\lambda_{-}(m)}$ from the sprinkler and the operating pressure of H_1 ; P_1 is

the average water application rate at the distance of d_1 from the sprinkler and the operating pressure of H_1 ; P_2 is the average water application rate at the distance of d_2 from the sprinkler and the operating pressure of H_1 . P_1 , P_2 , d_1 and d_2 can be known from the experimental data.

Similarly, the average water application rate at the distance of ρ from the aperature and the operating pressure of H can be

 $\frac{\rho}{\lambda_n(m)}$ from the sprinkler and the operating pressure of H_2 can be

calculated by Equation (13). H_2 is also the designated pressure at the sprinkler in experiment which is known.

$$P_{22}' = \frac{P_1' - P_2'}{d_1 - d_2} \frac{\rho}{\lambda_n(m)} + \frac{P_2' d_1 - P_1' d_2}{d_1 - d_2}$$
(13)

where, P_{22}' is the average water application rate at the distance of

 $\frac{\rho}{\lambda_n(m)}$ from the sprinkler and the operating pressure of H_2 ; P_1' is

the average water application rate at the distance of d_1 from the sprinkler and the operating pressure of H_2 ; P_2' is the average water application rate at the distance of d_2 from the sprinkler and the operating pressure of H_2 . P_1' , P_2' , d_1 and d_2 can be known from the experimental data.

Suppose that $H_1 \le H_n(m) \le H_2$, and combined with Equations (12) and (13), then the average water application rate at the

distance of $\frac{\rho}{\lambda_n(m)}$ from the sprinkler and the operating pressure

of $H_n(m)$ for the spot M_n' can be calculated by equation (14).

$$P_n'(m)_{average} = \frac{P_{11}' - P_{22}'}{H_1 - H_2} H_n(m) + \frac{P_{22}'H_1 - P_{11}'H_2}{H_1 - H_2}$$
(14)

Substituting Equation (14) into Equation (11), the instantaneous water application rate at the spot M_n' under sprinkler operating pressure of $H_n(m)$ on flat ground can be calculated using Equation (15).

$$P_{n}'(m) = \left(\frac{P_{11}' - P_{22}'}{H_{1} - H_{2}}H_{n}(m) + \frac{P_{22}'H_{1} - P_{11}'H_{2}}{H_{1} - H_{2}}\right)\frac{T_{n}'(m)}{t_{n}'(m)}$$
(15)

2.4 Estimating water distribution on sloping land

The water distribution for a single rotating sprinkler on sloping land can be obtained by the combination of each micro circular sector in Section 2.3. Additionally, when multiple sprinklers are used on sloping land, the water distribution can be regarded as the superposition of the water distribution of each sprinkler.

3 Model validations

3.1 Experimental design

To verify the model of water distribution for rotating sprinkler with pulsating pressure on a sloping land, an indoor experiment on water distribution of a single rotating sprinkler was conducted at the Irrigation Hydraulics Laboratory of Northwest A&F University, Yangling, China. Figure 4 shows the experimental setup, and Figure 5 shows the relationship among testing apparatus. The testing apparatus consisted of an automatic pressure control system, a stainless steel water tank, a sprinkler, a pressure sensor, height adjustable brackets, steel channels, and catch-cans.



1.Catch-can 2.Stainless steel channel 3.Variable-frequency control cabinet
4. Height adjustable bracket 5. Retaining plastic 6. Pressure sensor 7. Sprinkler
Figure 4 Experimental setup for the sprinkler water distribution on slopping land



Figure 5 Schematic diagram of relationship among testing apparatus

Pulsating pressure was controlled by an automatic pressure control system that consisted of a programmable logic controller (PLC), variable-frequency drive (VFD), and a centrifugal pump with an electric motor. Firstly, the program used for producing pulsating pressure was uploaded to the PLC to control the VFD, which modified the pump motor speed. Parameters of pulsating pressure (such as the function types, maximum pressure, minimum pressure, and function period) can be set in the program. Secondly, the electric motor speed of the centrifugal pump was adjusted by the VFD to produce the expected pulsating pressure. The pressure provided in this study was presented in Figure 6.



Figure 6 Pulsating pressure versus time

The Rainbird R5000 sprinkler was selected for the tests. The diameter of nozzle is 3.0 mm, the jet angle is 10°, and the manufacturer's recommended operating pressure ranges from 170 to 450 kPa. The pressure transducer was a Xi'an Xinmin model CYB in a range between 0 to 500 kPa at an accuracy of $\pm 0.1\%$. The pressure transducer wrapped in a plastic bag was installed at the nozzle inlet and connected to a data logger. Pressure was recorded at 5 s intervals by the data logger and the average pressure was calculated for each test.

The experimental terrain slope was artificially constructed by adjustable brackets and steel channels (with a length of 3.0 m and a width of 0.15 m). Steel channels were placed on the brackets, and the height of the brackets was calculated according to the terrain slope. Catch-cans (with a diameter of 10.6 cm and height of 14.0 cm) were placed in the steel channels and arranged at a spacing of 1.0 m×1.0 m. Water distributions of a single Rainbird R5000 sprinkler were measured at different pulsating pressures on the same slope. The slope was 10%, which is expressed as a ratio: increase in elevation over a horizontal distance. To keep the sprinkler operated at normal working pressure, average values of pulsating pressure were set to 250 kPa, 300 kPa, and 350 kPa according to the amplitude of pulsating pressure, which was 50 kPa. The function type of pulsating pressure was set trigonometric with a period of 18 s. There were totally three treatments, each treatment contained two tests, and the duration of each test is 1 h. In the first test, the sprinkler was installed on the top of the slope to record water distribution for downhill slope. In the second test, under exactly the same experimental conditions (the same pressure and height of sprinkler, etc.), the sprinkler was installed at the bottom of the slope to record water distribution for uphill slope. The water distribution for a single sprinkler on sloping land can be obtained by combining the water distribution measured on the uphill and downhill slopes. Each test was repeated three times, and the measured catch-can data were averaged.

Additionally, to provide the measured catch-can data for the model estimating water distribution at pulsating pressure on sloping land, water distributions of a single Rainbird R5000 sprinkler were also measured at constant pressures of 200 kPa, 250 kPa, 300 kPa, 350 kPa and 400 kPa on flat ground. A double radial leg of catch-cans was used such that at each distance from the sprinkler there were two cans, and the measured catch-can data from each pair were averaged. Catch-cans were placed at 0.106-m intervals in the radial direction, and were placed side-by-side. The distance between the center of the first

catch-can and the sprinkler location was 0.21 m. There were 150 catch-cans in the experiment for each radial leg, for a total of 300 catch-cans, and these were enough to extend beyond the wetted radius of tested sprinkler for all pressures. The weight of the water in each catch-can was measured using an electronic balance (Mettler Toledo, model XP10001S; 0.1 g accuracy) to calculate the water application rate.

3.2 Statistical analysis

Christiansen Uniformity Coefficient $(CU)^{[18]}$ and standard deviation (SD) and were used to evaluate sprinkler water application uniformity with the measured catch-can data. Scheduling coefficient (SC) represents the ratio of area receiving low water application rate to the average water application rate applied through the wetted area. *CU*, *SD* and *SC* are expressed in equation form as:

$$CU = 100 \left| \frac{\sum_{i=1}^{N} |P_i - \overline{P}|}{\sum_{i=1}^{N} P_i} \right|$$
(16)

$$SD = \sqrt{\frac{\sum_{i=1}^{N} (P_i - \overline{P})}{N}}$$
(17)

$$SC = \frac{\overline{P}}{Dlq}$$
(18)

where, P_i is the water application rate of individual catch-can, mm/h; \overline{P} is the average measured water application rate of all catch-cans, mm/h; D_{lq} is the mean value of the lowest one-quarter water application rate of all catch-cans, mm/h.

Catch3D is a mathematical model that can be used to analyze measured performance data of sprinklers in agriculture, including water application uniformity calculation^[19]. It was used for some

calculations of the results presented herein.

The accuracy of the model can be evaluated by mean bias error, root mean square error and the coefficient of determination, herein referred to as *MBE*, *RMSE* and $R^{2[20,21]}$. The closer the R^2 to 1 and the lower the *MBE* and the *RMSE*, indicating the higher accuracy of the model is.

MBE, *RMSE*, and R^2 are expressed in equation form as:

$$MBE = \frac{\sum_{i=1}^{N} P_{si} - P_{mi}}{N}$$
(19)

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (P_{si} - P_{mi})}$$
(20)

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} (P_{si} - P_{mi})^{2}}{\sum_{i=1}^{N} (P_{mi} - \overline{P_{m}})^{2}}$$
(21)

where, P_{si} is the simulated water application rate, mm/h; P_{mi} is the measured water application rate, mm/h; $\overline{P_m}$ is the average of measured water application rate, mm/h.

3.3 Comparison of simulated and measured water distribution

The absolute errors between simulated and measured water distribution for a single sprinkler at different pulsating pressure are shown in Figure 7, the slope was 10%, and the period of pulsating pressure was 18 s. The origin point (0, 0) was the location of the sprinkler. As shown in Figure 7, the simulated water distribution differs little from the measured data in the whole wetted area. At different pulsating pressure, the absolute errors of water distribution in different parts of the wetted area were basically less than 0.45 mm/h, and the maximum error was 1.8 mm/h, which was mainly distributed at the edge of the uphill slope.



Moreover, Table 1 shows the average value, CU and relative errors of water application rates in wetted area of a single sprinkler with pulsating pressure on sloping land. It can be seen that the value of average water application rate and CU between simulated and measured are very close to each other. The *MBE*, *RMSE* and R^2 are -0.046, 0.384, and 0.82, respectively, indicating the difference between simulated and measured water application rates in wetted area varied slightly. It can be concluded that the model could be used to predict water distribution of a single sprinkler with pulsating pressure on sloping land.

Fable 1	Average water application rate.	, CU and error o	f water application rates	in wetted area of	f single sprinkler
		,			

Average pressure	Amplitude	Average water application rate $/mm h^{-1}$			<i>CU</i> /%		
/kPa	/kPa	Simulated	Measured	Err/%	Simulated	Measured	Err/%
250	50	2.40	2.49	3.61	75.59	74.55	1.40
300	50	2.47	2.61	5.36	77.34	74.67	3.58
350	50	2.70	2.74	1.46	78.00	78.34	0.43

Moreover, in practical, multiple sprinklers are usually used on sloping land. To test the applicability of the model to multiple sprinklers used on sloping land, Table 2 shows the average water application rate, CU and relative errors of water application rate under different arrangements with pulsating pressure on sloping land. The relative errors of average water application rate were all

less than 3.1%, the relative errors of CU were all less than 3.9%, and the values of R^2 were all greater than 0.82, very close to 1. It proved that this model has a high accuracy in simulating the water distribution of multiple sprinklers with pulsating pressure on sloping land.

Table 2	Average water application rate.	CU and relative error	of water application rates in	wetted area of overlapped sprinkler
	in the age water applied to in the offer		or mater application rates in	in every an easier of errapped oprimier

Overlapping configuration		Average water application rate/mm h ⁻¹			CU/%			\mathbf{p}^2
Arrangement	Spacing	Simulated	Measured	Err/%	Simulated	Measured	Err/%	ĸ
Square	$1.0R_{0}$	7.44	7.22	3.05	74.08	73.49	0.80	0.82
Rectangular	$1.0R_0 \times 0.8R_0$	8.67	8.45	2.60	76.76	75.91	1.12	0.84
Equilateral triangle	$1.0R_{0}$	7.70	7.63	0.92	85.51	85.03	0.56	0.95
Equilateral triangle	$0.8R_{0}$	8.36	8.46	1.12	93.16	94.66	1.56	0.85
Equilateral triangle	$1.2R_0$	5.80	5.93	2.19	69.06	71.84	3.90	0.89

Note: R_0 is the throw radius for Rainbird R5000 sprinkler at the constant pressure of 300 kPa on flat ground.

4 Model applications

The above model was applied to study the effects of terrain slope, sprinkler arrangement, sprinkler spacing, and average pulsating pressure on water distribution for Rainbird R5000 sprinkler, which is important for sprinkler irrigation system design^[22,23]. The function type, period and amplitude of pulsating pressure were set as trigonometric, 18 s and 50 kPa in the model, respectively. The factors and levels for the simulation are listed in Table 3.

Table 3 Factors and levels for the simulation

Slope/%	Sprinkler spacing	Sprinkler arrangement	Average pulsating pressure/kPa
5	$0.6R_0$	Square	250
10	$0.8R_{0}$	Equilateral triangle	300
15	$1.0R_{0}$	Rectangular	350
20	$1.2R_0$	-	-
25	$1.4R_0$	-	-

Note: R_0 is the throw radius for Rainbird R5000 sprinkler at the constant pressure of 300 kPa on flat ground.

4.1 Effect of slope on water distribution

Table 4 lists the throw radius and the average water application rate for a single sprinkler on different slopes, and the average pulsating pressure was 300 kPa. As shown in Table 4, with the increase of terrain slope, the throw radius of uphill decreases, while the throw radius of downhill and the ratio of downhill throw to uphill throw increase. This is mainly because, when the slope increases, the water spray to the uphill will easily be hindered by the terrain slopes, which resulted in a shorter throw radius. While, when the water sprays to the downhill, it is not easily influenced by terrain slopes and has a longer throw radius. The throw radius directly affects the wetted area of the sprinkler. Due to the changes of throw radius, the wetted area decreases on uphill while increases on downhill. In the meantime, quantities of water sprayed from the sprinkler to uphill and downhill were the same. Therefore, the average water application rate increases for uphill, while decreases for downhill. As a result, the ratio of water application rate for downhill to uphill decreases with the increase of the slope, which indicated that the difference of water distribution between uphill and downhill increase and the uniformity of water distribution in the whole wetted area became worse. When the slope changed from 5% to 25%, CU declined from 69.06% to 63.76%.

As for multiple sprinklers, Figure 8 shows the water distribution patterns of three sprinklers with equilateral triangle arrangement on different slopes, and the average pulsating pressure was 300 kPa and sprinkler spacing was $1.0R_0$. Locations of the three sprinklers in Figure 8 were coordinates (0, 0), (10, 0) and (5, 8.66). As shown in Figure 8, water distribution patterns are similar on five slopes, that is, most of water is concentrated on the middle and lower parts of the wetted area, and a small quantity of water fell on the top of the wetted area. The area with low water application rate increase with the increase of the slope, and the value of SC are 1.90, 1.96, 2.14, 2.47 and 2.56 for slopes of 5%, 10%, 15%, 20% and 25%, respectively. Because the throw radius of uphill decreased with the increase of the slope, the water from two sprinklers located in coordinates (0, 0) and (10, 0) was difficult to reach the area on the top, and was more concentrated in the middle and edges of the wetted area, which makes the difference of water distribution in the wetted area increased. Therefore, the CU decreases with the increase of the slope. CU are 86.22%, 85.51%, 82.76%, 78.34% and 74.47% for slopes of 5%, 10%, 15%, 20% and 25%, respectively. The CU value for the slope of 20% was lower than 80%, which is the minimum CU recommended by many designers in practice^[24]. Consequently, it is recommended that the Rainbird R5000 sprinkler should be operated below the slope of 20% in practice.

Table 4 Throw radius and average water application rate for a single sprinkler on different slopes

Slope /%	Uphill throw/m	Downhill throw/m	Ratio of downhill throw to uphill throw	Average water application rate for uphill slope/mm h ⁻¹	Average water application rate for downhill slope/mm h ⁻¹	Ratio of water application rate for downhill slope to uphill slope	<i>CU</i> /%
5	8.87	10.77	1.21	2.33	2.19	0.94	69.06
10	8.91	11.54	1.30	2.39	2.13	0.89	68.82
15	8.90	11.89	1.34	2.44	2.10	0.86	68.38
20	8.68	12.95	1.49	2.47	2.02	0.82	67.98
25	8.10	14.32	1.77	2.56	1.90	0.74	63.76



4.2 Effect of sprinkler arrangement on water distribution

Water distribution patterns of sprinklers with three different arrangements by square $(1.0R_0 \times 1.0R_0)$, rectangular $(0.8R_0 \times 1.0R_0)$, and equilateral triangle $(1.0R_0)$ are shown in Figure 9, and the average pulsating pressure was 300 kPa, the slope was 10%. For square arrangement, the four sprinklers were located at the four vertices. Most water cumulated in the middle and lower part of the wetted area, where the application rate was larger than 8 mm/h. The application rates at the top or edges of wetted area were lower than 6 mm/h and took up 36.4% of the whole wetted area. Compared to the square arrangement, the water application rate of rectangular arrangement is more concentrated. The ratio of areas with low application rate (lower than 6 mm/h) to the whole wetted area decreased to 21.6%, the most obvious reduced area marked A in Figure 8a, while the left and right sides of the wetted area, which marked B and C, still existed in Figure 8b. This is because when sprinklers are arranged in rectangular, it shortens the distance

between the sprinklers on the uphill and downhill. Water sprayed from the downhill sprinklers could easily reach the areas near the uphill sprinklers, and the overall water application rate was improved. For equilateral triangle arrangement, the ratio of area with low application rate to the whole area decreased obviously, only took up 8.3%. Areas with low application rates only show on the top of the triangle. The reason is that the equilateral triangle arrangement shortens the distance between sprinkler on uphill to area B and area C. It is concluded that sprinklers with equilateral triangle arrangement have the best water distribution uniformity, with the CU of 85.51%. It is followed by the rectangular arrangement, with the CU of 76.76%. Sprinklers arranged in square has the worst water distribution uniformity and its CUaccounts for only 74.08%, which cannot meet the recommended CU values in practice. Therefore, it is recommended to use the equilateral triangle arrangement in the design of the sprinkler irrigation project with the pulsating pressure on sloping land.



4.3 Effect of sprinkler spacing on water distribution

The water application rate and its standard deviation under different sprinkler spacings are shown in Table 5. The slope was 10%, and the average pulsating pressure was 300 kPa. As sprinkler spacing increases, the maximum and minimum values of water application rate in the wetted area decrease gradually, and the minimum value decreases most obviously, which led to the range of the water application rate increases.

 Table 5
 Water application rate under different sprinkler

 spacing

Sprinkler spacing	Min to max/mm h^{-1}	Average/mm h ⁻¹	SD
$0.6R_0$	7.98-14.13	11.29	1.752
$0.8R_{0}$	7.63-10.17	8.36	0.711
$1.0R_0$	3.29-9.90	7.70	1.608
$1.2R_0$	2.13-9.70	5.80	2.190
$1.4R_{0}$	2.13-8.87	4.99	2.306

Analysis of the proportion of water application rate in different intervals with different sprinkler spacings is shown in Figure 10. When the sprinkler spacing is less than or equal to R_0 , the water application rate shows a high concentration. Values of water application rate are mainly distributed between average±2 mm/h. When the sprinkler spacings are $0.6R_0$, $0.8R_0$ and $1.0R_0$, the proportions of water application rate between average±2 mm/h are 80.6%, 96.3%, and 86.4%, respectively. When the sprinkler spacing is $1.2R_0$, the number of low water application rates increase obviously, and the proportions of water application rate near When sprinkler spacing average value reduced to 57.8%. increases to $1.4R_0$, the proportion of water application rates between 2.0-3.0 mm/h, which deviates far away from the average value, reaches more than 40%. It can also be seen in Table 5 that the value of standard deviation of the water application rates in wetted area increase sharply, which indicated that when the sprinkler spacing is greater than R₀, the dispersion degree of water application rates increase, and the uniformity of water distribution became worse. The CU values in five different spacings $(0.6R_0,$ $0.8R_0$, $1.0R_0$, $1.2R_0$ and $1.4R_0$) are 83.48%, 93.16%, 85.51%, 69.06% and 61.90%, respectively. This is because when sprinkler spacing was larger than sprinkler throw radius, water sprayed from sprinkler can hardly reach the area beyond the sprinkler throw radius, which lowers the water application rate in these areas, increases the heterogeneity of water distribution and decreases the CU value. When sprinkler spaced greater than $1.2R_0$, it cannot meet the requirements of sprinkler irrigation design. In conclusion, for the Rainbird R5000 series sprinkler, the sprinkler

spacing should be set between $0.8R_0$ and $1.0R_0$ when using pulsating pressure on sloping land.



Figure 10 Ratio of water application rate in different intervals to total data for sprinkler with different sprinkler spacing in wetted

area

4.4 Effect of pressure on water distribution

Figure 11 shows the water distributions for sprinklers with equilateral triangle arrangement under different average pulsating pressures, the sprinkler spacing was $1.0R_0$ and slope was 10%. Most water cumulated within the middle part of the wetted area, while the water application rate in the top area is relatively low. As average pulsating pressure increases, the area on the top with low water application rate decreases gradually, the value of SC were 2.34, 1.96 and 1.89 under the three water pressures of 250 kPa, 300 kPa and 350 kPa, respectively. Additionally, except for the area with low water application rate, the water supplication rate in other areas increased, and the proportion of water application rates around the average value in the wetted area increased. Standard deviations were 1.516, 1.435, and 1.393, respectively, and values of CU were 81.75%, 85.51%, and 86.08%, respectively. However, as average pulsating pressure increases, the effect of average pulsating pressure on the uniformity of water distribution is weakened. When water pressure increases from 250 to 300 kPa, CU value increases by 3.76%. When water pressure increases from 300 to 350 kPa, CU value only increases 0.57%, less than a quarter of the former. This is because the relationship between throw radius and average pulsating pressure is fractional power, and the exponent is 0.5, which means the increase rate of sprinkler throw radius is slower than that of water pressure, the area with low water application rate increases slowly with the increase of water pressure, and the effect of increasing water pressure on promoting the uniformity of water distribution gradually decrease. Considering the uniformity of water distribution and operating costs, RainBird R5000 sprinkler should be operated under the average pulsating pressure of 300 kPa.



5 Conclusions

Based on the water conservation principles, and the finite element idea, a mathematical model of water distribution under pulsating water pressure on sloping land was established, and its accuracy was experimentally verified. The model could accurately and reliably simulate the water distribution on sloping land when using pulsating pressure.

The effects of terrain slope, sprinkler arrangement, spacing, and average pulsating pressure on water distribution of sloping land under pulsating pressure conditions were analyzed. The gentler the terrain slope the higher the water distribution uniformity, and when the slope is greater than 20%, the water distribution uniformity cannot meet the sprinkler quality requirements. Sprinklers arranged in equilateral triangle have the best water distribution uniformity, with the uniformity coefficient (*CU*) of 85.51%, followed by the rectangular arrangement and square arrangement, with *CU* of 76.76% and 74.08%, respectively. For any increase in sprinkler spacing, the water application rate in the wetted area become scattered, and the uniformity of water distribution got worse with a faster trend. The uniformity of water distribution was increased with the average pulsating pressure, however, the increased extent was limited.

Considering the operational cost of the sprinkler systems and the water distribution uniformity, when the Rainbird R5000 sprinkler is used on sloping land with pulsating pressure, it is suggested that the sprinkler irrigation system should be arranged below the slope of 20%, with an average working pressure of 300 kPa, and the equilateral triangle arrangement is more suitable with sprinkler spacing of $0.8R_0$ - $1.0R_0$.

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[References]

- Darko R O, Yuan S Q, Liu J P, Yan H F, Zhu X Y. Overview of advances in improving uniformity and water use efficiency of sprinkler irrigation. Int J Agric & Biol Eng, 2017; 10(2): 1–15.
- [2] Montero J, Tarjuelo J, Carrián P. SIRIAS: A simulation model for sprinkler irrigation. Irrigation Science, 2001; 20(2): 73–84.
- [3] Zhang, L, Hui, X, Chen, J Y. Effect of terrain slope on water distribution and application uniformity for sprinkler irrigation. Int J Agric & Biol Eng, 2018; 11(3): 120–125.
- [4] Bishaw D, Olumama M. Evaluating the effect of operating pressure and riser height on irrigation water application under different wind conditions in Ethiopia. Asia Pac. J. Energy Environ, 2015; 2: 41–48.
- [5] Hui X, Yan H J, Zhang L, Chen J Y. A simplified method to improve

water distribution and application uniformity for sprinkler irrigation on sloping land: adjustment of riser orientation. Water Science & Technology Water Supply, 2021; 21(6): 2786–2798.

- [6] Xiang Q, Chen C, Wei Y. Application of sprinkler with variable range of spray in sloping land. Transactions of the CSAE, 2011; 27(8): 115–119. (in Chinese)
- [7] Tang Y, Luo Y, Yuan S Q, Liu J P. Variable range of precision spraying of lawn sprinkler for control strategy by variable pressure technology. Transactions of the CSAM, 2010; 41(12): 101–106. (in Chinese)
- [8] Chen C, Tang L, Gu Z, Tang Y. Hydraulic performance experiment of variable wetting sprinkler based on single-chip microcomputer. Transactions of the CSAE, 2014; 30(18): 116–122. (in Chinese)
- [9] Ge M, Wu P T, Zhu D, Zhang L, Gong X. Hydraulic performance of fixed spray plate sprinkler under dynamic water pressure. Transactions of the CSAM, 2015; 46: 27–33. (in Chinese)
- [10] Zhang L, Fu B Y, Ren N W, Huang Y. Effect of pulsating pressure on water distribution and application uniformity for sprinkler irrigation on sloping land. Water, 2019; 11(5): e913. doi: 10.3390/w11050913.
- [11] F dix-F dix J R, Salinas-Tapia H, Bautista-Capetillo C, Garcia-Aragon J, Burguete J, Playan E. A modified particle tracking velocimetry technique to characterize sprinkler irrigation drops. Irrigation Science, 2017; 35(6): 515–531.
- [12] Faria L C, Nörenberg B G, Colombo A, Dukes M D, Timm L C, Beskow S, et al. Irrigation distribution uniformity analysis on a lateral-move irrigation system. Irrigation Science, 2019; 37(2): 195–206.
- [13] Darko R O, Yuan S Q, Liu J P, Yan H F, Zhu X Y. Overview of advances in improving uniformity and water use efficiency of sprinkler irrigation. Int J Agric & Biol Eng, 2017; 10(2):1–15.
- [14] Zhang L, Merkley G P, Pinthong K. Assessing whole-field sprinkler irrigation application uniformity. Irrigation Science, 2013; 31(2): 87–105.
- [15] Zhang L, Fu B Y, Hui X. Simplified method for estimating throw radius of rotating sprinklers on sloping land. Irrigation Science, 2018; 36(7): 329–337.
- [16] Feng C D. Calculation of radius of throw for sprinkler. Drain Irrig Mach, 1984; 3(4): 35–38.
- [17] Feng C D. Theoretical study on sprinkler design. Drain Irrig Mach, 1986; 5(4): 16–20.
- [18] Christiansen J E. The uniformity of application of water by sprinkler systems. Agric. Eng., 1941; 22: 89–92.
- [19] Merkley G P. Catch3D Users' Guide; Civil and Environmental Engineering Department, Utah State University, Logan, UT, USA, 2004.
- [20] Yacoubi S, Zayani K, Zapata N, Zairi A, Slatni A, Salvador R, et al. Day and night time sprinkler irrigated tomato: Irrigation performance and crop yield. Biosyst Eng, 2010; 107(1): 25–35.
- [21] Ouazaa S, Burguete J, Paniagua M P, Salvador R, Zapata N. Simulating water distribution patterns for fixed spray plate sprinkler using the ballistic theory. Spanish Journal of Agricultural Research, 2014; 12(3): 850–863.
- [22] Moazed H, Bavi A, Boroomand-Nasab S, Naseri A, Albaji M. Effect of climate and hydraulic parameters on water uniformity coefficient in solid set system. Journal of Applied Sciences, 2010; 10: 1792–1796.
- [23] Li L H, Zhang X Y, Qiao X D, Liu G M. Analysis of the decrease of center pivot sprinkling system uniformity and its impact on maize yield. Int J Agric & Biol Eng, 2016; 9(4): 108–119.
- [24] Keller J, Bliesner R D. Sprinkler and trickle irrigation. An Avi Book Van Nostrand Reinhold Pun, New York, 1990; 651p.