Novel method for calculating flow depth in open channels combining the iteration theory and the curve-fitting technique

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Abstract: Calculation of critical depth in open channels or closed conduits is a prerequisite for efficient hydraulic design, operation, and maintenance of irrigation channels and drainage ditches. Determination of critical depth in the trapezoidal cross section is of particular significance as it is one of the most widely used channel sections throughout the world, while no closed-form analytical solutions exist. Based on the novel combined iteration-curve-fitting method, the existing equations were unified in the same function model, and two new equations were proposed for directly calculating critical depth in trapezoidal open channels. The maximum absolute relative errors of the two proposed equations are 0.004 94% and 0.165%, respectively, in wide application ranges. Comparison and evaluation of the proposed and existing equations for calculating critical depth in trapezoidal open channels were also presented. The introduction and application of the novel method could make the process of function model establishment much more efficient, which provides more insights into the hydraulic calculations of channels and ditches. Moreover, this paper provides reference for the problems related to the empirical equations of high-degree polynomial equations.

Keywords: critical depth, trapezoidal cross section, direct solutions, combined iteration-curve-fitting method, high-degree polynomial equation

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1 Introduction

Critical depth is the flow depth at a section where the flow is critical, which is one of the most significant hydraulic variables in design, operation, and maintenance of irrigation channels and drainage ditches[1]. The critical depth classifies channel flow as subcritical (mild), super-critical (steep), and critical conditions. Channel designers should ensure that flow remains sub-critical (high depth, low velocity) in the majority of its length. However, in short reaches, super-critical flow may be allowed. For this purpose, it is necessary to calculate the critical depth and find out the channel reaches under supercritical flow where appropriate measures could be implemented to prevent this adverse flow condition[2]. Even for open channels where critical flow may not occur at all, the critical depth is still calculated as the first step in dealing with most of the open channel flow problems[3].

The trapezoidal cross section is one of the most widely used open channel sections, whereas there are no analytical solutions to explicitly calculate critical depth in trapezoidal[4] and many other practical cross sections[5-9]. The critical depth in these cross sections is presently obtained by time-consuming trial-and-error procedures, chart methods with low accuracy, costly commercial computer programs, or explicit equations (if available). From the viewpoint of hydraulic engineers, it is preferable to have existing explicit equations to calculate hydraulic variables (e.g., critical depth in a channel) with both high accuracy and wide application range[5]; thus any efforts for deriving direct solutions meeting these requirements would be of practical importance. With respect to the calculation of critical depth in trapezoidal open channels, currently the most accurate existing formula was proposed by Vatankhah[8]; the maximum relative error is less than 6×10^{-6}% with one Newton-Raphson iteration calculation. Varandili et al.[10] developed an analytical model in a relatively complicated form to obtain calculation results with arbitrary accuracy. Despite the fact that various explicit equations are presently available for calculating critical depth in trapezoidal open channels[7,8,10-15], their accuracy, simplicity, and applicable range width have not proved optimum. Moreover, the determination of the function models exhibits some subjectivity and randomness.

Characteristic water depth (including critical, normal, contracted, and conjugate water depths)[16,17], as well as hydraulic jump[18]and flow velocity[19] in open channels were usually calculated with numerical analysis methods. Among various numerical analysis methods, the iterative algorithm was most commonly used[20]. For the iterative convergence procedure, an initial value is always needed to start the iterative calculation, and an appropriate function model should be chosen for optimal coefficient determination in the curve-fitting procedure. Therefore, a regression-based equation is always required to provide an approximation of the initial iteration value with certain accuracy. To the best of our knowledge, the selection of the function model that is most fitted to the inversion of a non-linear equation with no analytical solutions relies on the researchers' experience, and this process could be inefficient and time-consuming. Recently, some mathematical methods such as the Delta-perturbation method[21,22], the improved asymptote matching technique[16], Lagrange's inversion theorem[23,24], and Lambert W- function[25,26] were introduced to improve the formula determining efficiency of characteristic water depth calculations. With the continuous demand for farmland construction and channel design, more novel methods are bound to and must

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necessarily be applied to the field of hydraulics calculation.

The primary objective of this study was to introduce a novel method into the calculation of critical depth in trapezoidal open channels and derive new solutions with higher accuracy and wider application range, without sacrificing the simplicity of the formula form in a more efficient approach, which could provide technical reference for channel designers and operators.

2 Methods

2.1 Governing equation of critical depth in trapezoidal open channels

The geometric property of the trapezoidal cross section is shown in Figure 1. The critical flow condition in open channels can be described by the following relation[27]:

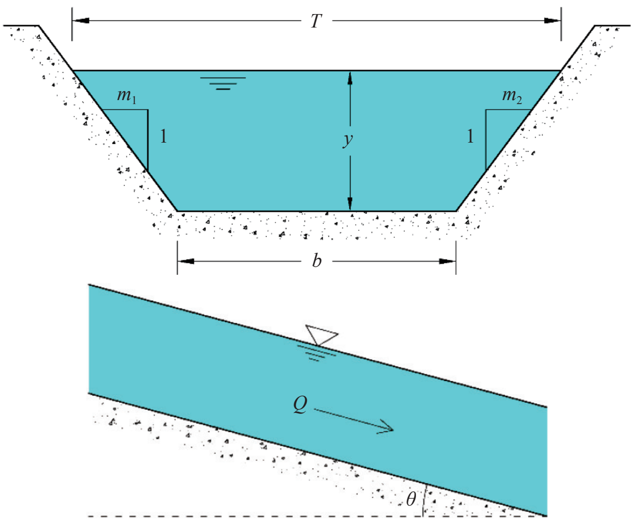

Figure 1 Cross section for a trapezoidal open channel

$$\frac{\alpha Q^2 T_c}{g A_c^3 \cos\theta} = 1 \tag{1}$$

where, the subscript "c" denotes the condition of the critical state of flow; α is the energy correction factor, non-dimensional; Q is the channel discharge, m^3/s; T_c is the width of the channel at the water surface when critical flow occurs, m; g is the gravitational acceleration, m/s^2; A_c is the cross section area of flow when critical flow occurs, m^2; θ is the angle of the channel bottom with the horizon, rad or (°).

According to the geometric property of the trapezoidal cross section, T_c and A_c in Equation (1) can be calculated with:

$$T_c = b + 2my_c \tag{2}$$

$$A_c = by_c + my_c^2 \tag{3}$$

where, b is the bottom width of the trapezoidal open channel, m; y_c is the critical depth of the channel when critical flow occurs, m; m is the mean value of two side slopes (m_1 and m_2) of the channel, non-dimensional.

The dimensionless critical depth η_c and the other three dimensionless variables λ_c, t_c, and ε are defined as below[8]:

$$\eta_c = my_c/b \tag{4}$$

$$\lambda_c = t_c^3 = 2\eta_c + 1 = T_c/b \tag{5}$$

$$\varepsilon = 4\left(\frac{\alpha m^3 Q^2}{gb^5 \cos\theta}\right)^{1/3} \tag{6}$$

where, η_c is the dimensionless critical depth, non-dimensional; λ_c is the dimensionless width of the channel at the water surface when critical flow occurs, non-dimensional; t_c is a dimensionless intermediate variable, non-dimensional; ε is the dimensionless flow discharge, non-dimensional.

Substituting for T_c, A_c from Equations (2)-(3) and t_c, ε from Equations (5)-(6) into Equation (1) finally yields the following governing equation: a sextic (6th degree) polynomial combining all the variables related to the critical flow condition into two dimensionless variables:

$$t_c^6 - \varepsilon t_c - 1 = 0 \tag{7}$$

2.2 Implementation of the combined iteration-curve-fitting method

Using the fixed-point iterative method, Equation (7) can be transformed into several iterative schemes as below:

$$t_{c_1} = \left(1 + \varepsilon t_{c_0}\right)^{1/6} \tag{8}$$

$$t_{c_2} = [1 + \varepsilon(1 + \varepsilon t_{c_0})^{1/6}]^{1/6} \tag{9}$$

$$t_{c_3} = \{1 + \varepsilon[1 + \varepsilon(1 + \varepsilon t_{c_0})^{1/6}]^{1/6}\}^{1/6} \tag{10}$$

If each constant and t_{c_0} in Equations (8-10) are treated as the object coefficients for curve fitting, three function models could be established as below:

$$t_c = (A + B\varepsilon^C)^D \tag{11}$$

$$t_c = [A + B\varepsilon(C + D\varepsilon^E)^F]^G \tag{12}$$

$$t_c = \{A + B\varepsilon[C + D\varepsilon(E + F\varepsilon^G)^H]^I\}^J \tag{13}$$

In order to obtain an explicit solution with a relatively wide range of flow conditions, $1.001 \le t_c \le 3.450$ ($0.001 \le \eta_c \le 20$) was applied for curve fitting[28]. Within this range, a total number of 2450 data points derived by Equation (7) with the same step of 0.001 were imported to the Curve Fitting Toolbox in MATLAB. Apply Equations (12) and (13) as the function models for coefficient optimization, and then set the corresponding initial coefficient values in Equations (9) and (10) as the start points, e.g., set $A=B=C=D=E=1$ and $F=G=1/6$ for Equation (12). The relative error (RE) and absolute error (AE) of the equations are calculated with:

$$\text{RE} = \frac{\text{AE}}{y_c^*} = \frac{y_c' - y_c^*}{y_c^*} \times 100\% = \frac{\eta_c' - \eta_c^*}{\eta_c^*} \times 100\% \tag{14}$$

where, y_c' is the calculated critical depth, m; y_c^* is the actual critical depth, m; η_c' is the calculated dimensionless critical depth, non-dimensional; η_c^* is the actual dimensionless critical depth, non-dimensional. RE is the relative error of y_c' and η_c', %; AE is the absolute error of y_c', m.

The average and maximum absolute RE were employed for the error analysis of the direct equations. Average absolute RE was calculated with the data points derived from the application range of η_c at intervals of 0.001; maximum absolute RE was calculated with the data point intervals as short as needed to approach to an accurate value. The values of the unknown coefficients were optimized until both the maximum and average absolute RE of the newly established equation have been the lowest.

3 Results and discussion

3.1 Proposed direct solution for critical depth in trapezoidal open channels

Two equations based on the function models of Equations (15) and (16) were finally derived:

$$\lambda_c = [1 + 0.9752\varepsilon(1.433 + 1.021\varepsilon)^{0.3175}]^{0.4566}, \quad \varepsilon \in [0.0032, 541.93] \tag{15}$$

$$\lambda_c = \{1 + 0.9849\varepsilon[1.16 + 1.25\varepsilon(0.835 + 0.607\varepsilon)^{0.271}]^{0.1664}\}^{0.4953}, \quad \varepsilon \in [0, 490.98] \tag{16}$$

The direct solutions developed in this study for calculating critical depth in trapezoidal open channels are summarized in Table 1.

Table 1 Direct solutions developed in this study for critical depth calculation in trapezoidal open channels

Step	Known value	Formula	Derivation
I	$\alpha, m, Q, g, b, \theta$	$\varepsilon = 4\left(\frac{\alpha m^3 Q^2}{g b^5 \cos\theta}\right)^{1/3}$	ε
II	ε	$\lambda_c = [1 + 0.9752\varepsilon(1.433 + 1.021\varepsilon)^{0.3175}]^{0.4566}$ $\lambda_c = \{1 + 0.9849\varepsilon[1.16 + 1.25\varepsilon(0.835 + 0.607\varepsilon)^{0.271}]^{0.1664}\}^{0.4953}$	λ_c
III	λ_c, b, m	$y_c = \frac{b}{2m}(\lambda_c - 1)$	y_c

3.2 Performance evaluation of the existing direct calculationequations

Several most accurate direct equations for calculating critical depth in trapezoidal open channels were selected for comparison with Equations (15) and (16) in terms of accuracy, application range, and simplicity. These equations happen to be able to be expressed in the form of the proposed function models in Equations (11)-(13). As shown in Figure 2 and Table 2, compared with the equation from Wang[15], Equation (15) is a direct solution of λ_c (sharing the same RE with η_c) with simplicity and 6.4-fold (the ratio of the maximum absolute REs of the two equations) higher precision using the same number of coefficients. Equation (16) is the most accurate among all the existing equations and exhibits a relatively wide application range; the maximum and average absolute REs are merely 0.004 94% and 0.002 69%, respectively, for different values of η_c in the practical range of [10^{-10}, 20.095]. Equation (16) is 24 times more accurate than the simplified equation by Wang[14], which is also in the three-iteration scheme in Equation (13) owing to the benefits of the introduced method. It is worth noting that Equation (16) provides approximations of y_c with absolute errors of less than 0.1 cm when y_c is as high as 20 m covering a totally wide practical range of flow conditions[28]. The proposed method evidenced the capacity of predicting critical depth in trapezoidal open channels with remarkable accuracy.

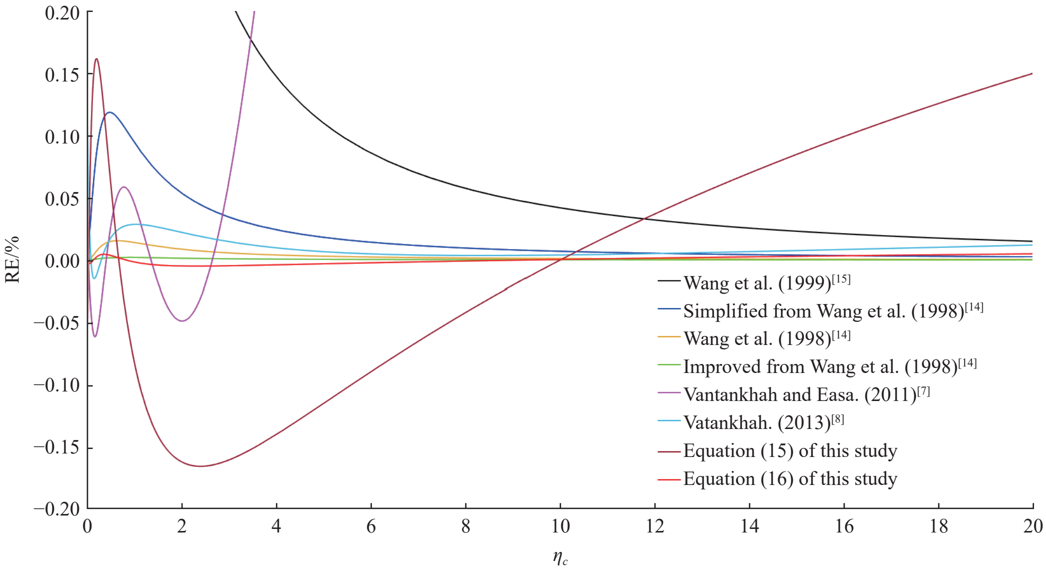

Figure 2 Relative error distribution on approximation of critical depth in trapezoidal open channels using the calculation equations in the form of the iterative schemes proposed in this study for $0.001 \leq \eta_c \leq 20$

Table 2 Comparison of the equations for calculating critical depth in trapezoidal open channels in the form of the iterative schemes proposed in this study

Source of formula	Form of formula	Recommended application range of η_c	Absolute RE	
			Maximum	Average
Wang et al.[15]	$\lambda_c = [1 + \varepsilon(1+\varepsilon)^{0.2}]^{0.5}$	(0, +∞)	1.048%	/
Simplified from Wang[14]	$\lambda_c = [1 + \varepsilon(1 + \varepsilon(1+\varepsilon)^{0.2})^{1/6}]^{0.5}$	(0, +∞)	0.118%	/
Wang[14]	$\lambda_c = \{1 + \varepsilon[1 + \varepsilon(1 + \varepsilon(1+\varepsilon)^{0.2})^{1/6}]^{1/6}\}^{0.5}$	[10^{-12}, +∞)	0.0154%	/
Improved from Wang[14]	$\lambda_c = \{1 + \varepsilon[1 + \varepsilon(1 + \varepsilon(1 + \varepsilon(1+\varepsilon)^{0.2})^{1/6})^{1/6}]^{1/6}\}^{0.5}$	[10^{-11}, +∞)	0.002 14%	/
Vatankhah and Easa[7]	$\lambda_c = 1 + 0.5\varepsilon(1 + 0.2722\varepsilon^{1.041})^{-0.339}$	(0, 2.999]	0.0611%	0.0336%
Vatankhah[8]	$\lambda_c = [1 + 1.161\varepsilon(1 + 0.666\varepsilon^{1.041})^{0.374}]^{0.432}$	[0.062, 41.987]	0.0286%	0.0152%
Varandili et al.[10]	$\lambda_c = \{1 + \varepsilon[1 + \varepsilon(1 + \varepsilon(1 + \varepsilon(1 + \cdots)^{1/6})^{1/6})^{1/6}]^{1/6}\}^{0.5}$	(0, +∞)	/	/
Equation (15)	$\lambda_c = [1 + 0.9752\varepsilon(1.433 + 1.021\varepsilon)^{0.3175}]^{0.4566}$	[0.0008, 21.351]	0.165%	0.0915%
Equation (16)	$\lambda_c = \{1 + 0.9848\varepsilon[1.16 + 1.25\varepsilon(0.607\varepsilon + 0.835)^{0.271}]^{0.1664}\}^{0.4953}$	[10^{-10}, 20.095]	0.004 94%	0.002 69%

Note: The formula proposed by Vatankhah and Easa[7] was expressed in the form of Equation (12).

3.3 Insights from the novel combined iteration-curve-fitting method

The proposed combined iteration-curve-fitting method has both theoretical basis and practical value. There are quantities of dimensionless governing equations without analytical solutions for the calculation of different characteristic depths (e.g., critical depth, normal depth, conjugate depth) in various cross sections. Some regression-based equations were established with the function models selected according to experience with a certain randomness. Among these governing equations, some high-degree polynomial equations can be transformed into simple iterative schemes that provide perfect function models. The related channel cross sections include not only trapezoidal, but also various parabola-shaped channels[29,30]. The iterative scheme derived with several times of iterations totally ensures the accuracy of the equation. These iteration-based functions themselves provide good approximation of the initial values of coefficients and ensure better potential of convergence in the coefficient optimization procedure, thus improving the efficiency for establishing new equations.

Compared with Delta-perturbation method[21,22], the improved asymptote matching technique[16], Lagrange's inversion theorem[23,24], and Lambert W‐function[25,26], the method combining the iteration theory and the curve-fitting technique method proposed in this paper establishes the iterative schemes concisely from the original governing equation, and gives full play to the advantages of curve fitting for parameter optimization. The theory provides an effective approach for seeking appropriate formula forms of curve fitting and explains why the presently available equations for calculating critical depth in trapezoidal sections are in similar forms. Moreover, the method is expected to provide references for the calculation of characteristic water depths of parabola-shaped cross sections[29,30], as well as other problems relying on the direct solutions of non-linear equations.

4 Practical application

To illustrate the application of the equations available in this study and their reliability, critical depths calculation with the application of Equations (15) and (16) and equations present in existing studies (Table 2) were conducted for two actual man-made concrete-lined open channels (Table 3). The critical depths were calculated with the hypotheses that α=1.0, cosθ=1, g=9.7964 m/s^2.

Table 3 Comparison of equations for calculating critical depth in trapezoidal open channels in the form of the iterative schemes proposed in this study

Equations	m	Q/(m^3·s^{-1})	b/m	ε	y_c'/m	RE/%	AE/mm
Accurate value calculated with the trial method					1.641 784 904	0	0
Equation (15)					1.641 819 467	2.1×10^{-3}	0.035
Equation (16)					1.641 812 765	1.7×10^{-3}	0.028
Wang et al.[15]					1.655 418 141	0.8300	13.630
Simplified from Wang[14]	1.25	28.37	3.05	3.389	1.643 640 618	0.1100	1.860
Wang[14]					1.642 037 915	0.0150	0.250
Improved from Wang[14]					1.641 819 408	0.0021	0.035
Vatankhah and Easa[7]					1.642 664 110	0.0540	0.880
Vatankhah[8]					1.642 186 969	0.0240	0.400
Accurate value calculated with the trial method					1.599 885 703	0	0
Equation (15)					1.599 239 168	–0.040	–0.650
Equation (16)					1.599 886 233	3.6×10^{-5}	0.000 58
Wang et al.[15]					1.611 698 593	0.740	11.810
Simplified from Wang[14]	1.25	23.35	2.44	4.317	1.601 568 746	0.110	1.680
Wang[14]					1.600 125 876	0.015	0.240
Improved from Wang[14]					1.599 919 984	2.1×10^{-3}	0.034
Vatankhah and Easa[7]					1.600 815 206	0.058	0.930
Vatankhah[8]					1.600 323 154	0.027	0.440

As listed in Table 3, the REs of Equation (16) are merely 1.7×10^{-3}% and 3.6×10^{-5}%, which are the most accurate among all the equations. The AEs of Equation (16) are below 0.1 mm, indicating that the accuracy could absolutely meet the practical application demand. With fewer times of power function calculation, the REs of Equation (15) are of the same orders of magnitude or more accurate compared with all the existing equations apart from the formula improved from Wang[14]. It should be mentioned that excessively pursuing accuracy is not necessary, as the construction of open channels only requires that the AE is below 1 mm or even 1 cm in most cases. As exhibited in the two application cases, the optimization of the parameters of the basic iteration function using the curve-fitting technique could greatly improve the accuracy of the equation and the efficiency of the equation selection procedure.

5 Conclusions

This study has considered a dimensionless governing equation of critical flow in trapezoidal open channels that includes only two variables. Through the application of the novel combined iteration-curve-fitting method, two direct calculation equations were derived and the existing most accurate equations were classified into iteration-based forms. The maximum absolute REs of the two proposed equations are respectively 0.004 94% and 0.165% over wide practical ranges. The results of accuracy evaluation and practical application indicated that one of the proposed equations is preferable to previously presented solutions in terms of accuracy. The novel method greatly improves the efficiency of establishment of the direct solution of high-degree polynomial equations, which is expected to be a useful tool in the design, operation, and maintenance of open channels and related hydraulic structures.

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